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MULTI-SEASONALITY IN THE TBATS MODEL USING DEMAND FOR ELECTRIC ENERGY AS A CASE STUDY

Abstract. *Many researchers are familiar with time series forecasting, yet they struggle with specific types of data that require a suitable model of analysis. One such type of data might be seasonality. However, one has to note that most popular models (e.g. ARIMA and exponential smoothing) only account for one seasonality. This article presents the capabilities of the TBATS model which has no seasonality constraints, making it possible to create detailed, long-term forecasts.*

Keywords: *time series, forecasting, seasonality, TBATS, ARIMA, exponential smoothing, Fourier series.*

JEL Classifications: C53, C32, Q47

1. Introduction

Prediction of phenomena is as old as humanity itself; however, the scientific approach thereto began only with the development of higher mathematics and the derivatives of its teachings. One type of prediction, a time series forecasting, seeks to predict the scale of a phenomenon for a given period based on existing numerical data recorded at regular intervals.Whether one wants to predict stock prices, unemployment rates, or temperatures, it is important to choose the model that best describes the phenomenon in the past, and that will best predict it in the future. Depending on its nature, a time series may consist of the following components: trends, seasonal movements, cyclical movements, and irregular

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fluctuations. Time series are often analyzed using aggregate data in order to obtain a single seasonality and an adequate forecast period. For example, monthly data for the next year may be predicted on the basis of monthly data from some dozen years prior; and data for the next few weeks may be predicted on the basis of data from some dozen weeks or months prior.This article will present the capabilities of one of the more advanced TBATS models which account for seasonality using specific time series, e.g. with hourly data collected over a period of several years, and thus generates a medium-term forecast with the specificity of a short-term forecast. For this analysis of time series with multi-seasonality, we used hourly data on the demand for electric energy in Poland from a 14-year period which made it possible to observe and use three seasonalities. The body of this article is divided into three sections approximating the model and the forecasts made therewith, starting from the most general (with one monthly seasonality), and ending with detailed forecasts that account for the triple seasonality resulting from the use of hourly data from the entire research period.

2. Literature review

The demand for the electric energy and the related issues, such as setting up energy tariffs, building autonomous energy systems, Internet of energy, regulation of energy, integration of energy systems and the like represents the very important and timely questions humanity faces nowadays. Therefore, they are often tackled in the research literature that employs the mathematical modelling and econometric tools in attempt to answer these issues(Štreimikienė et al., 2016).

One of the factors in determining correct forecasts for a given phenomenon is selection of an appropriate model. Our selection of the TBATS model may seem peculiar given the number of other available models((Box, et al., 2016),(Zeliaś, et al., 2016),(Armstrong, 2001),(Brockwell & Davis, 1996)). The most frequently used models are ARMA/ARIMA/SARIMA, (Box & Jenkins, 1976),(Lee & Ko, 2011),(de Andrade & da Silva, 2009),(Pappas, et al., 2008),(Chen, et al., 1995); and exponential smoothing(Taylor, 2003),(Hyndman, et al., 2008).Using the ARIMA and exponential smoothing models is perfectly appropriate as long as they are not used for very complicated time series. We have also used these models in our previous works, as they were sufficient for the investigation of other phenomena(Brożyna, et al., 2016),yet forecasts of more complicated time series require more advanced models that employ Bayesian procedures (Cottet & Smith, 2003),Gaussian processes (Blum & Riedmiller, 2013),ant colony optimization (Dongxiao, et al., 2010), and many other methods (Zhou, et al., 2006),(Taylor, et al., 2006),(Küçükdeniz, 2010)depending on the specifics of the data. One such specific is multi-seasonality. The majority of these models are only good for modeling time series with one or two seasonalities. The solution to this problem is the TBATS model, introduced a few years ago (De Livera, et al., 2011). By applying this model, we can simultaneously account for many seasonalities

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occurring in a given time series and resulting in a detailed forecast for a longer period of time.

3. Data Profile

To demonstrate the capabilities of the TBATS model with regard to the seasonality of forecasted phenomena, we use data on the demand for electric energy from Polskie Sieci Elektroenergetyczne (Polish Electroenergy Networks, an Internet service), recorded from January 2002 through October 2015. The values of the time series observed have been presented in megawatts, and with 15-minute intervals, giving it a length of nearly half a million observations. In order to prepare and compare forecasts, the data were aggregated into hourly data (N=121248), daily data (N=5052), and monthly data (N=166) by selecting the maximum value for the period; moreover, the data are expressed in megawatts [GW] to increase legibility. The aggregation of data by selecting maximum values was justified by the need to research the energy system's maximum load. Selection of mean values would have been more justified from a purely statistical point of view, but would not have reflected reality. Selection of maximum values results in a loss of minimal values as the aggregation period expands, which in turn has an influence on the remaining descriptive statistics[1] (Table 1), the shapes of the graph (Figure 1, Figure 2, Figure 3), and forecasts.

Table 1. Descriptive statistics of maximum energy demand in Poland [GW]. Source: The authors' own research.

Type of aggregation	N	Min [GW]	1st Quarter [GW]	Median [GW]	Mean [GW]	3rd Quarter [GW]	Max [GW]
Hourly	121248	9.75	15.30	17.39	17.52	19.70	25.84
Daily	5052	12.75	18.28	20.14	19.99	21.97	25.84
Monthly	166	15.96	20.37	21.93	21.73	23.30	25.84

[1]Polskie Sieci Elektroenergetyczne, http://www.pse.pl/index.php?dzid=77 (02.11.2015)

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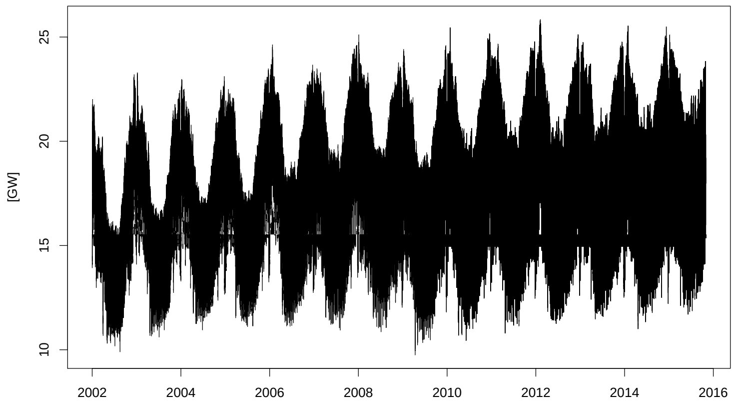

Figure 1. Maximum hourly demand for energy in Poland.
Source: The authors' own research.

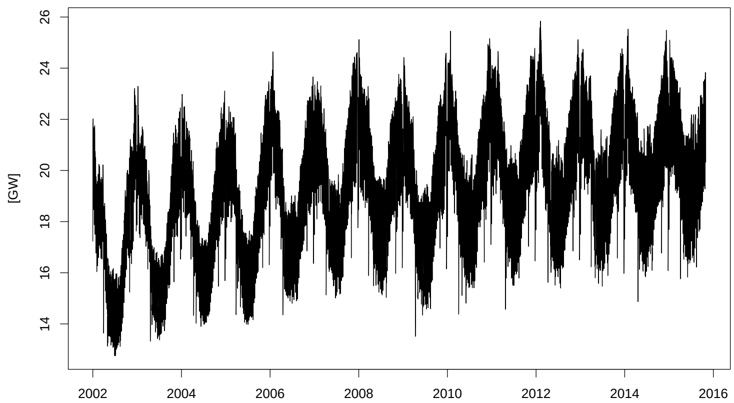

Figure 2. Maximum daily demand for energy in Poland.
Source: The authors' own research.

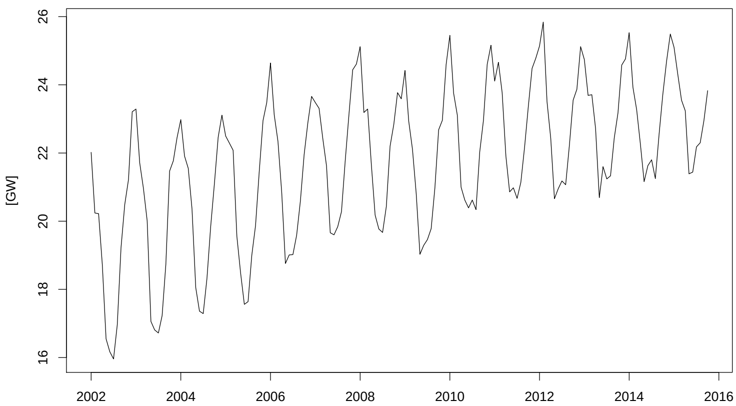

Figure 3. Maximum monthly demand for energy in Poland.
Source: The authors' own research.

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The three figures above show annual seasonality, with the lowest demand for energy in summer, and the highest in winter. The greater demand for energy in the winter can be explained by the shorter days (which call for longer operation of all types of lighting), and lower temperatures (which call for heating).
Further seasonalities are illustrated by the figure below of the hourly demand for energy in a sample month (Figure 4). This figure shows weekly and daily seasonality, with five business days and two weekend days. Additional analysis of source data shows that increased energy usage occurs from dawn till dusk, and is at its lowest after midnight.

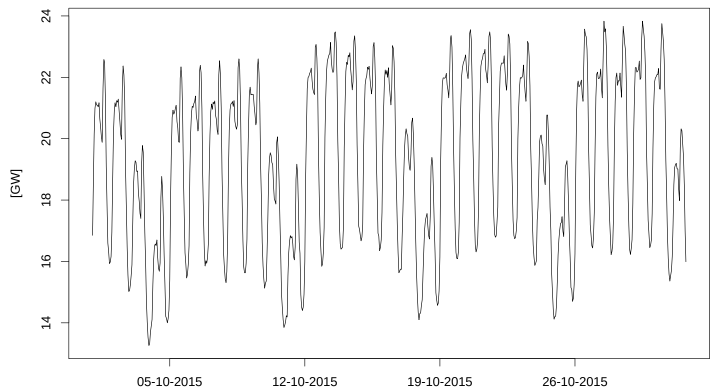

Figure 4. Maximum hourly demand for energy in Poland in October 2015. Source: The authors' own research.

The descriptive statistics in
Table **1** indicate a significant difference (up to 10 GW) between the minimum and maximum demand for energy. Considering that the data only concern a fairly short period of 14-years, these are significant differences that testify to the rapid increase in energy demand.

4. Models and forecasts

Figure 1 - Figure 4 allow us to draw general conclusions about the trends and seasonality of the time series studied. Regardless of the frequency of the data analyzed, energy use increased during the beginning period, and stabilized in the final years. Furthermore, the frequency of the time series affected the occurrence of yearly, weekly, and daily seasonality. The seasonalities occurring in time series limit the selection of prognostic models. Popular models such as ARIMA, exponential smoothing, and homologous period trend estimation may be used for smaller amounts of monthly data, and only when there is one annual seasonality. But these models do not work well for long series with overlapping seasonality, such as in the case of the daily and hourly data presented here, which contain two and three seasonalities, respectively. About ten years ago, Taylor proposed the double seasonal Holt-Winters model(Taylor, 2006),but it did not

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account for series with more than 2 seasonalities. The solution to the seasonality problem may be the TBATS model, which was introduced in 2011 and accounts for multiseasonality (**T**rigonometric, **B**ox-Cox transform, **A**RMA errors, **T**rend, and **S**easonal components)(De Livera, et al., 2011),with arguments
TBATS(ω, $\{p, q\}$, φ, $\{<m_1, k_1>, <m_2, k_2>,...,<m_T , k_T>\}$)
where:
ω is a Box-Cox transformation (Box & Cox, 1964),
p, q are ARMA parameters (Whittle, 1951),(Box & Jenkins, 1976),(Brockwell & Davis, 1996),
φ is a damping parameter (Gardner & McKenzie, 1985),(Snyder, 2006),
m_1,...,m_T are seasonal periods,
k_1,...,$k_T$are the number of Fourier series pairs (West & Harrison, 1997),(Harvey, 1989).

The model can be written as:

$$y_t^{(\omega)} = l_{t-1} + \varphi b_{t-1} + \sum_{i=1}^{T} s_{t-1}^{(i)} + \alpha d_t$$
$$b_t = b_{t-1} + \beta d_t$$
$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$
$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t$$
$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t$$
$$\lambda_j^{(i)} = \frac{2\pi j}{m_i}$$

where:
$i=1,\dots,T$
d_t is an ARMA (p,q) process,
α, β, γ_1 and γ_2 are smoothing parameters,
ℓ_0 is the initial level,
and b_0 is slope value.

Forecasts using this model will be presented starting from the most general monthly data, then daily data, and then hourly data. Forecast errors will be determined on the basis of the most commonly used measurements(Bratu, 2012):

Mean **A**bsolute **E**rror $MAE = \frac{\sum_{t=1}^{n} |e_t|}{n}$

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Mean **S**quared **E**rror $MSE = \frac{\sum_{t=1}^{n} e_t^2}{n-1}$

Root **M**ean **S**quared **E**rror $RMSE = \sqrt{\mathrm{MSE}}$

Mean **A**bsolute **P**ercentage **E**rror $MAPE = \frac{\sum_{t=1}^{n} \left| \frac{e_t}{y_t} \right|}{n} \cdot 100\%$

as well as less commonly-used measurements (Hyndman & Koehler, 2006),(Gardner, 1985):

Mean **E**rror $ME = \frac{\sum_{t=1}^{n} e_t}{n}$

Mean **P**ercentage **E**rror $MPE = \frac{\sum_{t=1}^{n} \frac{e_t}{y_t}}{n} \cdot 100\%$

Mean **A**bsolute **S**caled **E**rror $MASE = \frac{\sum_{t=1}^{n} |e_t|}{\frac{n}{n-m} \sum_{t=m+1}^{n} |y_t - y_{t-m}|}$

Auto**c**orrelation **f**unction of errors at lag **1** $ACF1 = \frac{\sum_{t=1}^{n-1} (e_t - ME) \cdot (e_{t+1} - ME)}{\sum_{t=1}^{n} (e_t - ME)^2}$

where:
e_t is the error $e_t = y_t - y_t^*$
y_t is the actual value,
y_t^* is the forecast value,
m is the seasonal period (Hyndman & Koehler, 2006).

When comparing ME and MAE errors or MPE and MAPE errors, we can tell whether the values in the forecast are systematically lower or higher than the observed values, and if they are multidirectional. Furthermore, analysis for MSE errors may indicate the occurrence of abnormally large errors; whereas extreme errors are indicated by significant differences between MAE and RMSE.

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4.1. Single seasonality in monthly data forecasts

Decomposition of the series containing monthly data (Figure 5) confirms the additive nature of the data. The trend is clearly growing, with a minimum value of 19.5 GW at the beginning, and 23.63 GW in the last observation from October 2015. Seasonal fluctuations range from -2.29 to 2.54 GW.

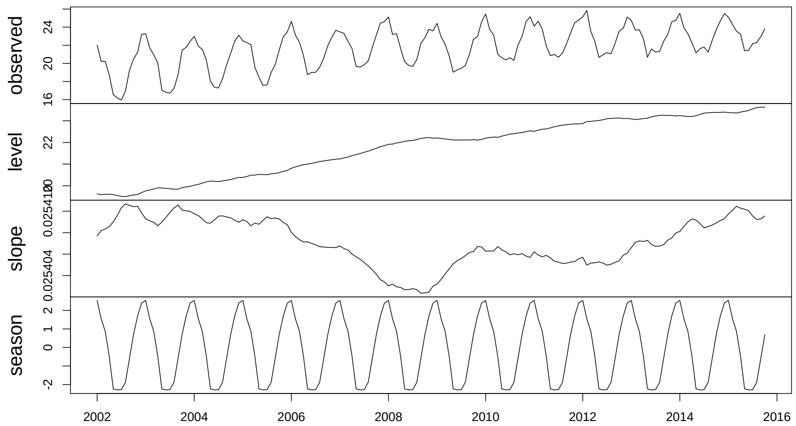

Figure 5. Decomposition of the time series with monthly data for energy demand. Source: The authors' own research.

The seasonality of the time series examined is also confirmed by Figure 6, which shows a higher demand for energy in winter months, and a lower demand in summer months. In analyzing the years on the right side of the graph, we can see that in most cases there is an increased demand for energy in each consecutive year. Yet what is interesting is that the graphs become flat, which means that in recent years, the demand for energy in the summer period is growing faster than that in the winter period.

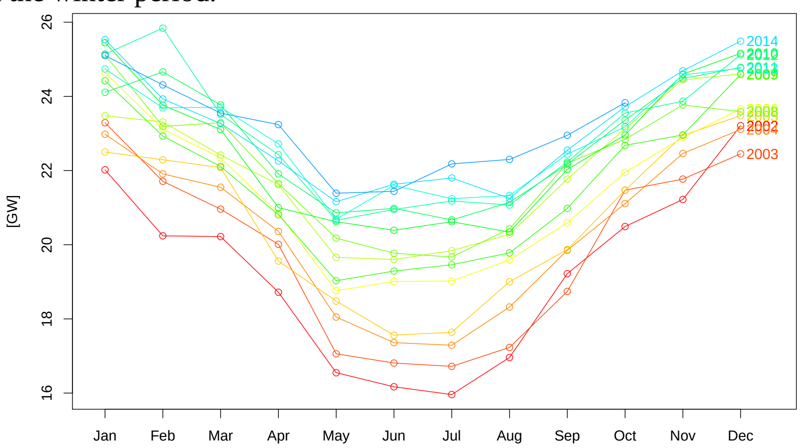

Figure 6. Seasonality of monthly data for energy demand in Poland. Source: The authors' own research.

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The parameters of the TBATS model (1, {1,2}, 1, {<12,5>}) are best-suited to the data. Forecasts for the next 12 months using this model are presented in Figure 7.

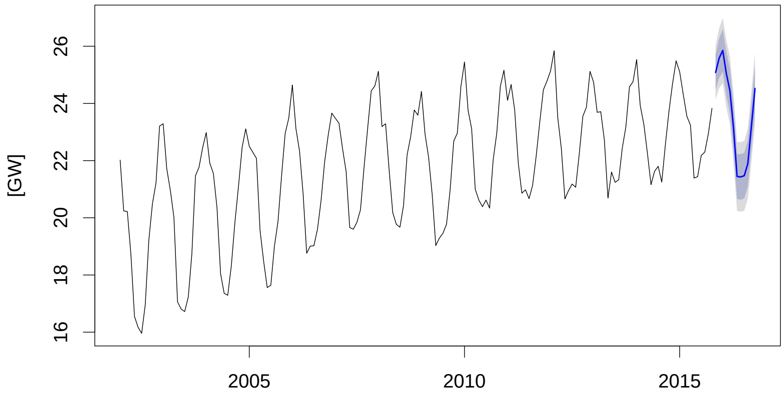

Figure 7. Forecast for the maximum monthly energy demand in Poland.
Source: The authors' own research.

In this model, Box-Cox transformation equals 1 (doing nothing), ARMA (1,2) errors are present, the damping parameter equals 1 (doing nothing), and 5 Fourier pairs have a period of m=12. This model can be written as:

$$y_t = l_{t-1} + b_{t-1} + s_{t-1} + \alpha d_t$$
$$b_t = b_{t-1} + \beta d_t$$
$$s_t = \sum_{j=1}^{k} s_{j,t}$$
$$s_{j,t} = s_{j,t-1} \cos\left(\frac{2\pi jt}{12}\right) + s^*_{j,t-1} \sin\left(\frac{2\pi jt}{12}\right) + \gamma_1 d_t$$
$$s^*_{j,t} = -s_{j,t-1} \sin\left(\frac{2\pi jt}{12}\right) + s^*_{j,t-1} \cos\left(\frac{2\pi jt}{12}\right) + \gamma_2 d_t$$

where d_t is an ARMA (*1,2*) process, and α, β, γ_1 and γ_2 are smoothing parameters. Here the seasonality has been handled with 12 parameters (the ten initial values for $s_{j,0}$ and $s^*_{j,0}$ and the two smoothing parameters γ_1 and γ_2). The total number of degrees of freedom is 20 (the other 8 come from the two smoothing parameters α and β, the four ARMA parameters, and the initial level and slope values ℓ_0 and b_0).
These forecasts conform to the trend and amplitude of the series studied. The confidence intervals are small and average ±0.80 GW for an 80% confidence interval, and ±1.22 GW for a 95% confidence interval. The errors of this model are small, and are as follows:

ME	RMSE	MAE	MPE	MAPE	MASE	ACF1

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-0.012130	0.482416	0.384576	-0.104051	1.786988	0.630386	0.000141

These arguments illustrate the suitability of this model to make realistic forecasts.

4.2. Double seasonality in forecasts of daily data

Analysis of a time series for daily data characterized by two seasonalities (yearly and weekly) is discussed in a separate article (Brożyna, et al., 2016),and this subchapter only contains the fragments most important for comparison with other types of seasonalities.
The decomposition result shown in Figure 8 confirms the yearly seasonality (season 2); however, the frequency of the first seasonal component (season 1) in relation to the length of the time series makes identification possible only after a graph for a shorter period (e.g. one month) has been analyzed. Taken as an example, the first seasonal component for October 2015 (Figure 9) clearly shows a weekly cycle.

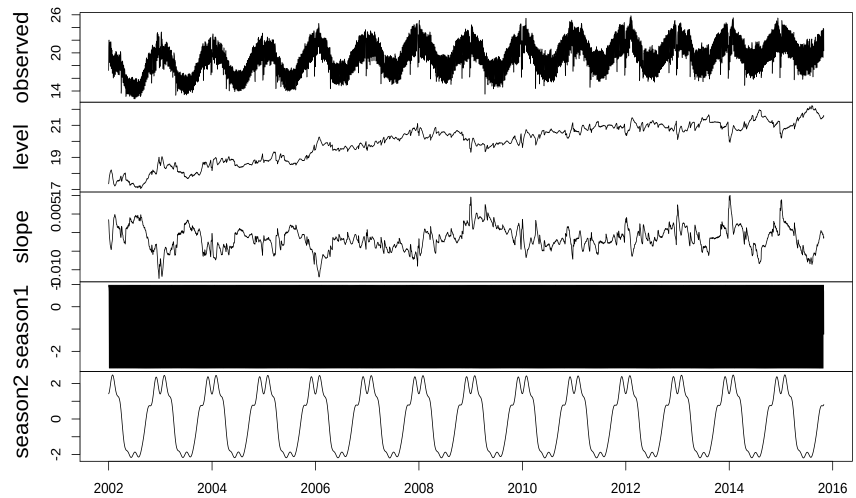

Figure 8. Decomposition of the time series with daily data on energy demand. Source: Brożyna, J., Mentel, G. and Szetela, B., 2016. Influence of double seasonality on economic forecasts on the example of energy demand. *Journal of International Studies*, 9(3), pp. 9-20.

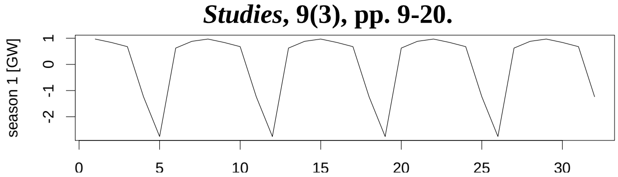

Figure 9. The first seasonal component after the decomposition of daily data. Source: ibid.

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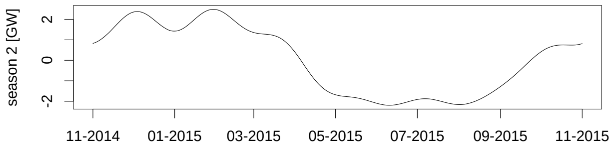

Figure 10. The second seasonal component after the decomposition of daily data. Source: ibid.

Analysis of yearly seasonality for daily data (Figure 8, Figure 10) confirms the conclusions of the analysis of monthly data. Furthermore, it allows us to locate the decline in maximum demand for energy during December and January, and July and August. For daily data, TBATS is the best-suited model (1, {5,5}, 0.986, {<7,3>, <365.25,7>}), whose forecasts for the following year are shown in Figure 11 and Figure 12.

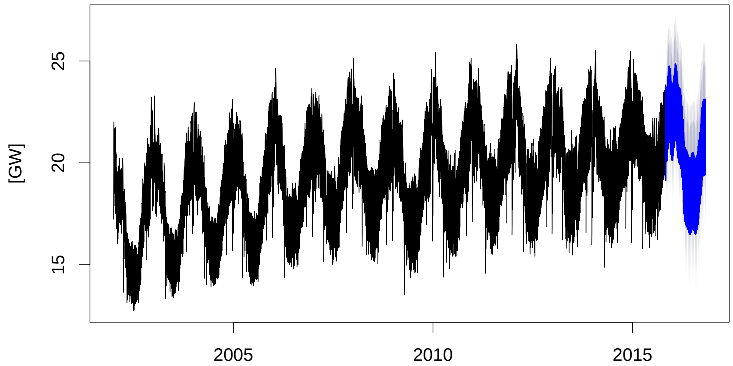

Figure 11. Historical data and forecasts for the maximum daily energy demand. Source: ibid.

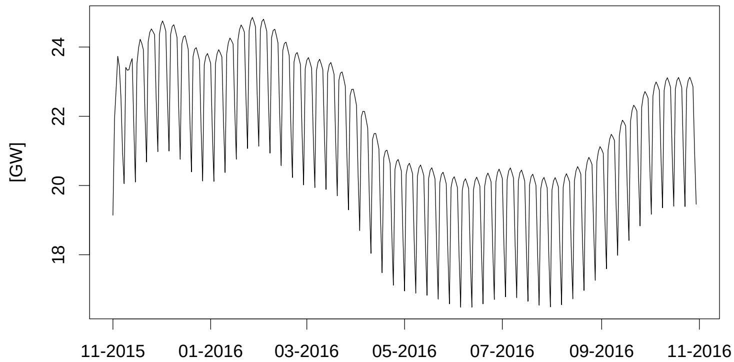

Figure 12. Forecast for the maximum daily energy demand in Poland. Source: ibid.

In this model, Box-Cox transformation equals 1 (doing nothing), ARMA (5,5) errors are present, the damping parameter equals 0.986 (essentially doing nothing), 3 Fourier pairs have a period of m_1=7 (weekly), and 7 Fourier pairs have a period

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of m_2=365.25 (annual). This model, adjusted as such, is characterized by the following errors:

ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
0.025845	0.758416	0.423351	-0.01815	2.235313	0.301647	-0.00655

In Figure 12 we see that the forecasts retain weekly seasonality with less energy demand on weekends, shown in Figure 9; as well as yearly seasonality with a decline in demand at the turn of the year and during summer, shown in Figure 10. Forecasts of daily data are characterized by confidence intervals more than double those for forecasts of monthly data; however, these values do not deviate significantly from the forecasts (as far as confidence intervals are concerned), and are on average ±1.81 GW for an 80% confidence interval, and ±2.77 GW for a 95% confidence interval.

4.3. Triple seasonality in forecasts of hourly data

Analysis of a time series containing hourly data from a 14-year period is not only complicated, but time-consuming. The data presented here include three seasonalities: yearly, weekly, and daily. The seasonal components after the decomposition of the time series (Figure 13, Figure 14, Figure 15) are not as regular as the monthly and daily data, and change in subsequent years.

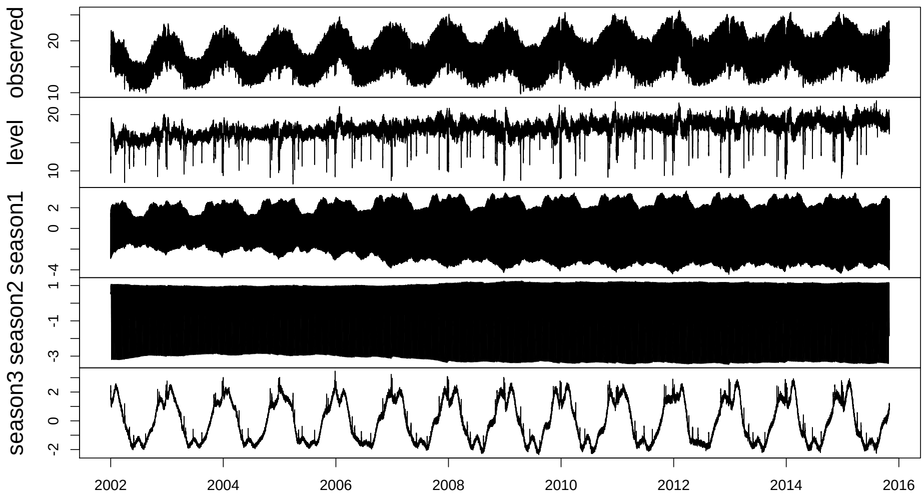

Figure 13. Decomposition of the time series with hourly data on energy demand. Source: The authors' own research.

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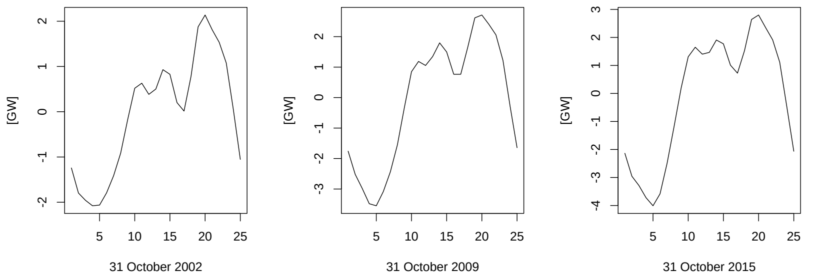

Figure 14. The first seasonal component after the decomposition of hourly data. Source: The authors' own research.

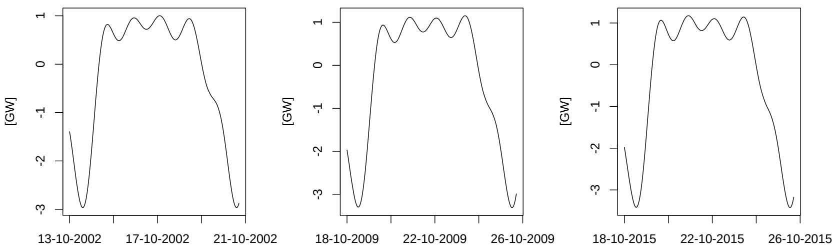

Figure 15. The second seasonal component after the decomposition of hourly data. Source: The authors' own research.

For hourly data, the best model is TBATS (1, {0,0}, 0.978, {<24,6>, <168,5>, <8766,5>}), with the following errors:

ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
0.000023	0.348757	0.255832	-0.029391	1.499548	0.090513	-0.001679

The difference between the MAE and RMSE errors is negligible, the mean absolute percentage error (MAPE) is about 1.5%, and the MASE error indicates that the accuracy of the forecasts is greater within the learning data set than for the average naive forecast. These criteria and the small ACF1 autocorrelation between errors illustrate the suitability of this model to the data.

In this model, Box-Cox transformation equals 1 (doing nothing), ARMA (0,0) errors are present, the damping parameter equals 0.978 (essentially doing nothing), 6 Fourier pairs have a period of m_1=24 (daily), 5 Fourier pairs have a period of m_2=168 (weekly), and 5 Fourier pairs have a period of m_3=8766 (annual) which can be written as:

$$y_t = l_{t-1} + b_{t-1} + s_{t-1}^{(1)} + s_{t-1}^{(2)} + s_{t-1}^{(3)} + \alpha d_t$$
$$b_t = b_{t-1} + \beta d_t$$
$$s_t^{(1)} = \sum_{j=1}^{6} s_{j,t}^{(1)}$$

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$$s_{j,t}^{(1)} = s_{j,t-1}\cos\left(\frac{2\pi jt}{24}\right) + s_{j,t-1}^{*}\sin\left(\frac{2\pi jt}{24}\right) + \gamma_1^{(1)} d_t$$
$$s_{j,t}^{*(1)} = -s_{j,t-1}\sin\left(\frac{2\pi jt}{24}\right) + s_{j,t-1}^{*}\cos\left(\frac{2\pi jt}{24}\right) + \gamma_2^{(1)} d_t$$
$$s_t^{(1)} = \sum_{j=1}^{5} s_{j,t}^{(2)}$$
$$s_{j,t}^{(2)} = s_{j,t-1}\cos\left(\frac{2\pi jt}{168}\right) + s_{j,t-1}^{*}\sin\left(\frac{2\pi jt}{168}\right) + \gamma_1^{(2)} d_t$$
$$s_{j,t}^{*(2)} = -s_{j,t-1}\sin\left(\frac{2\pi jt}{168}\right) + s_{j,t-1}^{*}\cos\left(\frac{2\pi jt}{168}\right) + \gamma_2^{(2)} d_t$$
$$s_t^{(2)} = \sum_{j=1}^{5} s_{j,t}^{(3)}$$
$$s_{j,t}^{(3)} = s_{j,t-1}\cos\left(\frac{2\pi jt}{8766}\right) + s_{j,t-1}^{*}\sin\left(\frac{2\pi jt}{8766}\right) + \gamma_1^{(3)} d_t$$
$$s_{j,t}^{*(3)} = -s_{j,t-1}\sin\left(\frac{2\pi jt}{8766}\right) + s_{j,t-1}^{*}\cos\left(\frac{2\pi jt}{8766}\right) + \gamma_2^{(3)} d_t$$

where d_t is an ARMA process $(0,0)$, and α, β, $\gamma_1^{(1)}$, $\gamma_2^{(1)}$, $\gamma_1^{(2)}$, $\gamma_2^{(2)}$, $\gamma_1^{(3)}$ and $\gamma_2^{(3)}$ are smoothing parameters. Seasonality was approximated by 14 parameters (12 initial values for $s^{(1)}_{j,0}$ and $s^{*(1)}_{j,0}$, and two smoothing parameters, $\gamma_1^{(1)}$ and $\gamma_2^{(1)}$). Approximation of weekly and yearly seasonalities consists of 12 parameters (10 initial values for $s^{(2)}_{j,0}$ and $s^{*(2)}_{j,0}$, and two smoothing parameters,$\gamma_1^{(2)}$ and $\gamma_2^{(2)}$, for weekly seasonality; and 12 initial values for $s^{(3)}_{j,0}$ and $s^{*(3)}_{j,0}$, and two smoothing parameters, $\gamma_1^{(3)}$ and $\gamma_2^{(3)}$, for yearly seasonality). Taking into account the two smoothing parameters α and β,, four ARMA process parameters, as well as initial values and slope values ℓ_0 and b_0, the total number of degrees of freedom is 46.

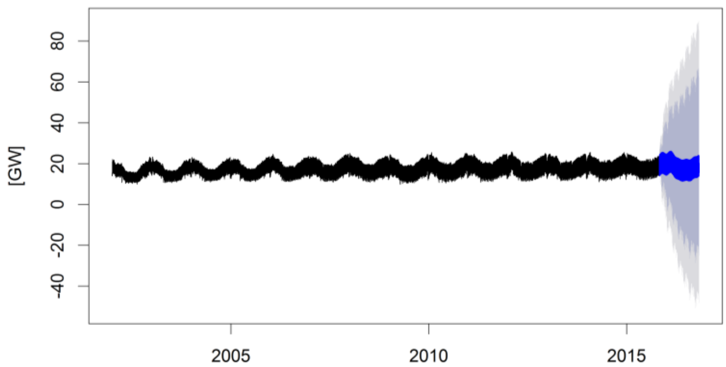

Figure 16. Historical data and forecasts for the maximum hourly energy demand. Source: The authors' own research.

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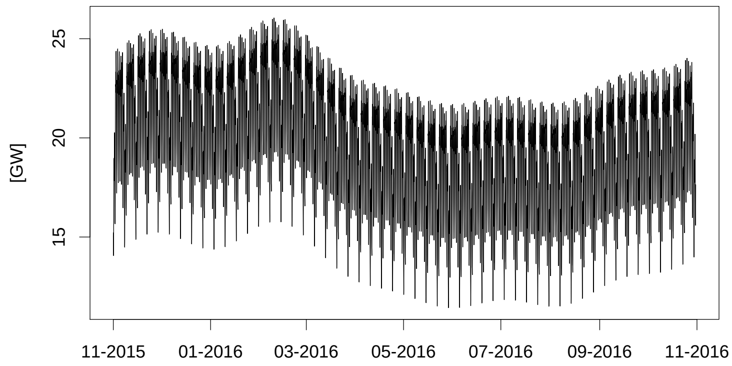

Figure 17. Forecast for the maximum hourly energy demand in Poland.
Source: The authors' own research.

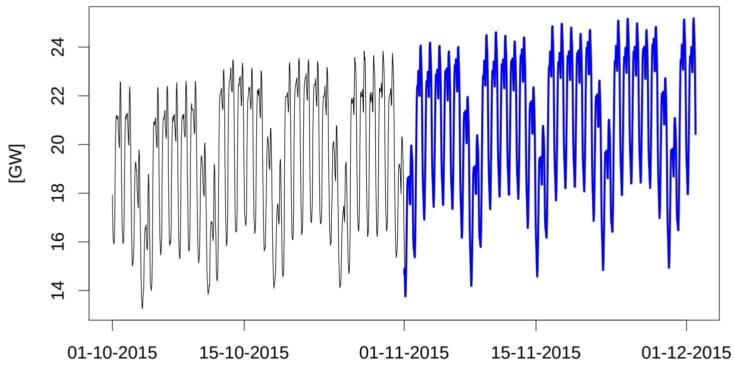

Figure 18. The last month of historical data, and the forecast for the maximum hourly energy demand in Poland for November 2015.
Source: The authors' own research.

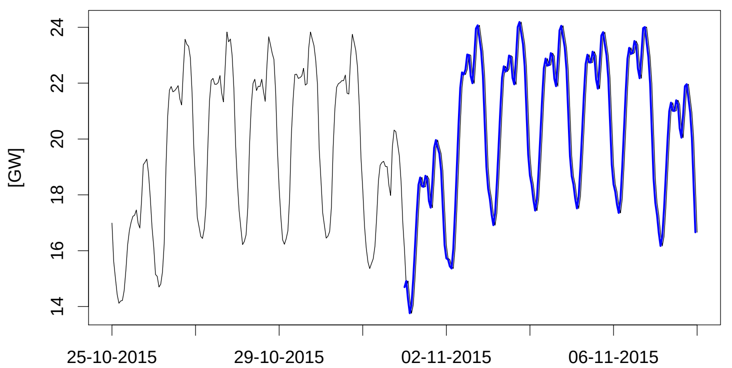

Figure 19. The last week of historical data, and the forecast for the maximum hourly energy demand in Poland for the first week of November 2015.
Source: The authors' own research.

Figure 16 shows that the forecast based on the hourly data retains the trend of the time series, but unlike forecasts made on the basis of monthly or daily data, its

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confidence intervals have a wide range. This is not a strange phenomenon, as forecasts usually include dozens of forward observations; yet in this case, they number 8776, which, in spite of the same forecasting period for monthly and daily data (1 year), results in an accumulation of "uncertainty." Analysis of successive graphs showing only the forecast for the next year (Figure 17), as well as historical data and forecasts for one month (Figure 18) and one week (Figure 19), indicates that the model is best-suited to the data. The forecasts retained all three seasonalities, making them more realistic for both shorter and longer time periods.

5. Conclusion

In an effort to forecast a given phenomenon, researchers aggregate data so as to obtain a time series that suits the horizon of the forecast; for example, hourly data for a short-term forecast, or monthly data for a long-term forecast. This is often the result of using basic models of ARIMA and exponential smoothing, whose limitations lead mainly to an increase in the forecast horizon, and a decrease in its level of detail. The solution to this problem is the TBATS model, which allows detailed forecasting of time series for longer periods of time. Using specific data on the demand for electric energy, this article shows long-term forecasts containing one, two, and three seasonalities. What is important is that the seasonality of data is not an obstacle for the TBATS model, and even renders the forecast period independent of the frequency of the data in the time series. It was thus possible to create e.g. a yearly forecast containing detailed hourly data. Of course, data for a distant future period may contain more errors, but key is the fact that when simultaneously accounting for all three seasonalities in a time series, we right away receive short-, medium-, and long-term forecasts.

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